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NOTATION

a_1, a_2 = dimensionless velocities of the dispersion waves
a_v = area available for heat transfer between gas and solid phases in a packed bed
c = concentration (normalized with respect to unit concentration)
C_f, c_s = heat capacities of fluid and solid, respectively
d = particle diameter
D = dispersion coefficient
D_a, D_r = axial and radial dispersion coefficients, respectively
P_e = Peclet number = ud/D
t = time variable
T = temperature
u = interstitial velocity
U_h = heat transfer coefficient between gas and solid
v = volume of a cell (in cell model)
x = distance coordinate
y = dimensionless distance coordinate
z = dimensionless time coordinate

Greek Letters

ϵ = voidage of the packed bed
η, λ, μ = see Equations (5) and (6)
γ = length of a cell
ρ_f, ρ_s = densities of fluid and solid, respectively
θ = residence time in a cell
τ = fraction of the total residence time which the fluid spends in the plug flow section of a cell

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Conjugate Unsteady Heat Transfer From a Droplet in Creeping Flow

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The phenomenon of unsteady heat transfer from a spherical droplet or solid particle moving slowly in a different fluid is analyzed, for the case in which the thermal resistances of the dispersed and continuous phases are comparable. The effect of the Peclet number (over the range $0 \le Pe \le 1{,}000$) and of the internal circulation of the droplet on the development of temperature fields is investigated. The energy equations for the interior and exterior of the droplet are solved by the finite-difference method of alternating directions. The results are compared with a number of previously published approximate models.

SCOPE

The problem of unsteady heat transfer from droplets moving in another immiscible fluid is important for a number of engineering applications. The majority of theoretical studies on this problem have been carried out on the assumption that most of the thermal resistance is concentrated either in the dispersed phase or in the continuous medium. But in many cases (for example, in direct-contact heat exchangers), the physical properties of the two phases are similar, and their thermal resistances are comparable in magnitude. The available solutions pertain to limiting cases of either very low ($Pe = 0$) or very high ($Pe \rightarrow \infty$) Peclet numbers. For $Pe = 0$, an analytic solution was obtained by Cooper (1977). For high Peclet numbers, two competing approximate models have been published. The first of these assumes that a thin thermal boundary layer exists at both sides of the droplet surface (Levich et al. 1965, Chao 1969). In the second model (Elzinga and Banchero 1959, Brounshtein et al. 1970) the ideas of Kronig and Brink (1950) on a highly developed circulation within the droplet are applied, but the boundary conditions at the droplet surface make allowance for the heat transfer resistance in the continuous phase.

There is no satisfactory solution for the case of comparable phase resistances at intermediate values of Pe. Nevertheless, this region is important for the analysis of heat transfer from small droplets moving at $Re \lesssim 1$, since the Prandtl numbers for liquids are of the order of 10-1,000, and hence, $Pe = Re \cdot Pr \lesssim 1{,}000$.

The purpose of the present article is the study of the physics of interphase transport on the basis of a correct numerical solution of complete equations for energy transport. In particular, the effect of the Peclet number (over the range $0 \le Pe \le 1{,}000$) and of the internal circulation on the development of temperature fields with time in both phases is investigated. The limits of applicability of existing approximate solutions of the given problem were also determined.

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CONCLUSIONS AND SIGNIFICANCE

An implicit finite-difference method was developed for the study of conjugate heat transfer from a slowly moving droplet (with and without internal circulation) to an immiscible fluid at intermediate Peclet numbers ($Pe \leq 1{,}000$). Numerical experiment allowed the study of the physical pattern of transient temperature fields both within and outside of the droplet. The particle cooling period can be essentially divided into two stages. In the first, heat is transferred primarily by conduction, and the isotherms have an almost spherical shape. In the second stage, convective heat transfer predominates. The effect of the internal circulation manifests itself during the second stage, and the Nusselt number at high Pe oscillates in time about its asymptotic value. Also during this stage, the internal isotherms take shapes close to streamlines, as predicted by the model of Kronig and Brink (1950).

A comparative analysis of the applicability of different approximate models was carried out for the case of contact of phases with similar physical properties, when the thermal resistance inside and outside the droplet are of comparable magnitudes. It was found that the external heat transfer to the particle at $Pe \lesssim 100\text{-}1000$ can be regarded as quasi-steady after a short initial time lapse. Hence, the average particle temperature for large Pe can be calculated from approximate models (Brounshtein et al. 1970, for a droplet with internal circulation; Gröber et al. 1961, for a solid sphere). In these, the external Nusselt number is assumed to be constant and equal to its quasi-steady value. During the short initial time from the start of the process, the heat transfer from a droplet with an internal circulation can be satisfactorily described by the boundary-layer model of Levich et al. (1965) and Chao (1969).

At moderate Peclet numbers ($Pe \lesssim 10 \div 100$), the values of the relaxation time of the external temperature field and the characteristic time of cooling (heating) of a particle are of the same order of magnitude. For this reason, the asymptotic value of the external Nusselt number is lower than its corresponding quasi-steady value. The existing approximate models were found not suitable for this range of Pe.

The phenomenon of heat transfer from rigid spherical particles, droplets or bubbles of one fluid moving in another fluid is important for a number of chemical engineering processes (e.g., direct-contact heat exchangers and fluidized-bed reactors). Many authors have proposed empirical correlations or theoretical solutions to describe heat transfer between a single droplet and a continuous phase. See, for example, the review of Sideman (1966) and the recently published book of Clift et al. (1978).

Theoretical solutions of the problem must be based on a knowledge of the flow fields in each phase. In the present work, we discuss the case of steady creeping flow ($Re \leq 1$) about a circulating droplet, when the non-dimensional stream functions of the flow inside and outside the liquid sphere are described by the Hadamard-Rybczinski equations

$$\psi_1 = -\frac{r^2(1 - r^2)}{4(1 + K)} \sin^2\theta \qquad (r \leq 1) \quad (1)$$

and

$$\psi_2 = \frac{1}{4}\left(2r^2 - \frac{2 + 3K}{1 + K} r + \frac{K}{1 + K} \cdot \frac{1}{r}\right) \sin^2\theta \;\; (r \geq 1) \quad (2)$$

The majority of the theoretical studies available deal with the two limiting cases, for which the rate of heat transfer is controlled by the thermal resistance either of the dispersed phase or of the continuous phase. The former case is known in the literature as "the internal problem," and the latter as "the external problem" or as "the model of a completely mixed droplet." In both these approaches, it is postulated that the temperature gradients in the noncontrolling phase may be neglected and that the interface temperature does not change with time.

The *internal* heat transfer problem for a rigid sphere or stagnation droplet ($Pe = 0$) was solved analytically by Newman (1931). The case at the other extreme—full internal circulation and high Peclet numbers ($Pe \to \infty$)—was first analyzed by Kronig and Brink (1950) and later by Heertjes et al. (1954), Elzinga and Banchero (1959), Brignell (1975) and others.

Johns and Beckmann (1966), Watada et al. (1970) and Wellek et al. (1970) published numerical solutions for the intermediate range of Pe. Some of the general conclusions that may be drawn from these works are described below. The unsteady Nusselt number Nu_{in} decreases from very high values at the start of the process and approaches some asymptotic value of Nu^*_{in} at long contact times ($\tau \to \infty$). At $Pe > 80(1 + K)$, the local Nusselt number $Nu_{in}(\tau)$ oscillates with a damping amplitude. The period of these oscillations correlates with the circulation time of the liquid over a closed path within the droplet. The asymptotic values of the Nusselt number increase monotonically from $Nu^*_{in} = 6.58$ at $Pe = 0$ (the solution of Newman, 1931) to $Nu^*_{in} = 17.9$[*] at $Pe \to \infty$ (Kronig and Brink 1950). The characteristic time of cooling[†] or heating of the particle, t_e, may be defined as the time required for the dimensionless mean droplet temperature $\overline{Z}_1 = (\overline{T}_1 - T_\infty)/(T_0 - T_\infty)$ to reduce to $1/e$ of its initial value. According to the Newman solution,

$$t_{e_1} \simeq 0.056 \; a^2/\alpha \quad (\text{at } Pe = 0), \qquad (3)$$

while the Kronig-Brink solution for $Pe \to \infty$ yields

$$t_{e_1} \simeq 0.022 \; a^2/\alpha \qquad (4)$$

The *external* problem of heat transfer has been examined for the so-called quasi-steady case. In this approach, the external Nusselt number $Nu_{ext}(\tau)$ is assumed to be constant with time and equal to Nu_{st}, which is the steady-state Nusselt number corresponding to the case of constant surface temperature. Values of Nu_{st} have been calculated by a number of authors. Levich (1962) obtained solutions at $Pe \to \infty$, by using the model of the thin thermal boundary layer

$$Nu_{st} = 0.99 \; Pe^{1/3} \qquad (5)$$

for a rigid sphere, and

$$Nu_{st} = 0.65\sqrt{Pe/(1 + K)} \qquad (6)$$

for a droplet with internal circulation. At low and intermediate Peclet numbers ($Pe \lesssim 1{,}000$) and different values of K, the steady problem was solved numerically by Abramzon and Fishbein (1977). For the quasi-steady case, the characteristic cooling time may be easily calculated from the heat balance equation for the droplet

$$\frac{4}{3}\pi a^3 \rho_1 C_1 \frac{d\overline{T}_1}{dt} = -4\pi a^2 \frac{Nu_{ext}\lambda_2}{2a}(\overline{T}_1 - T_\infty) \qquad (7)$$

Integration of (7) with the initial conditions $\overline{T}_1 = T_0$ at $t = 0$ gives

$$t_{e_2} = \frac{2a^2\rho_1 C_1}{3Nu_{st}\lambda_2} \qquad (8)$$

The unsteady external heat transfer from a sphere with a constant surface temperature has been studied by Konopliv and Sparrow (1972), Chao (1969) and Abramzon et al. (1976). They found that the external Nusselt number changes as $Nu_{ext} \simeq$

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[*] Elzinga and Banchero (1959) suggested a value of 17.66 for this case.

[†] From the mathematical point of view, the problem of cooling and heating of a droplet are identical. Hence, we limit our discussion to the cooling problem.

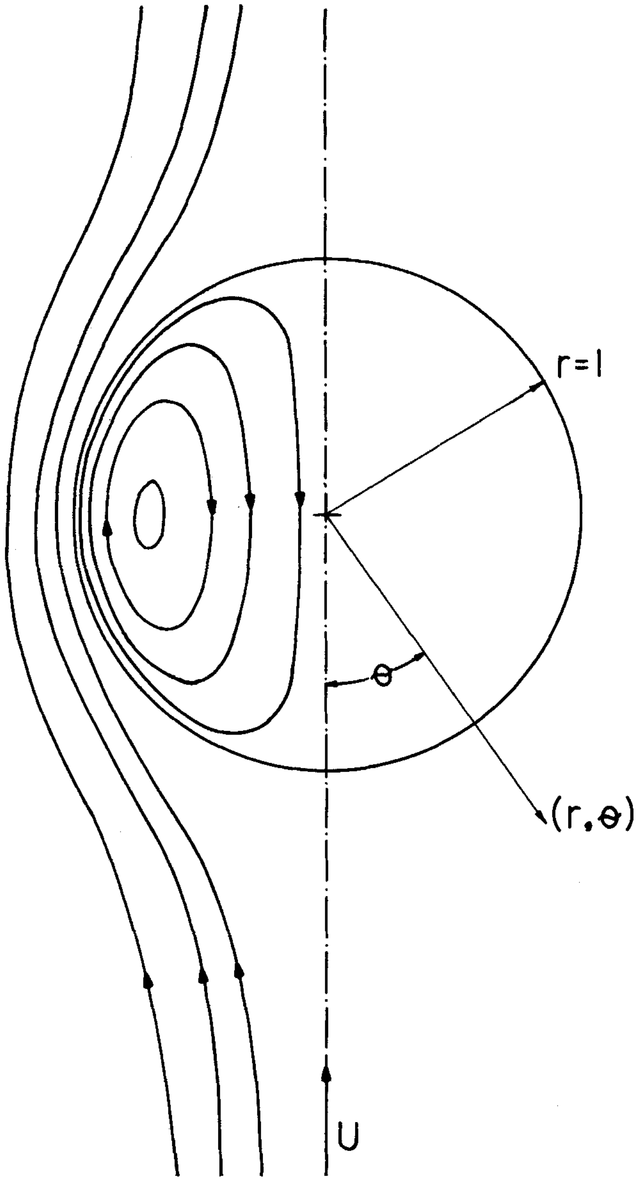

Figure 1. Coordinate system and flow pattern in and around a liquid droplet at low Reynolds numbers.

$2/\sqrt{\pi\tau}$ for short times, and approaches its steady-state value of Nu_{st} after a relaxation time τ_r. This time, which may also be called the "formation time of the external thermal field," depends on Pe and K. For the rigid particle at $Pe = 1; 10; 100; 1000$ the times required for Nu_{ext} to come within 5% of the steady value are $\tau_r \simeq 10; 1; 0.15$ and 0.02, respectively. For a rigid sphere at high Pe, $\tau_r \simeq 1/Pe^{2/3}$ while for a circulating droplet $\tau_r \simeq 1/Pe$.

Hence, the quasi-steady assumption may be justified if $\tau_r << \tau_{e_2}$. At large Pe, this condition may be rewritten as $Nu_{st} >> \rho_2 C_2/\rho_1 C_1$. To estimate the contribution of each phase to the total heat transfer resistance, we compare the two characteristic cooling times, t_{e_1} and t_{e_2}. These times are calculated on the assumption that the transfer is governed solely either by the external or by the internal resistance. It is clear that the external resistance is negligible, if $t_{e_1} >> t_{e_2}$, $t_{e_1} << t_{e_2}$, then the internal resistance may be neglected. Finally, the thermal resistances in the two phases are comparable, if t_{e_1} and t_{e_2} are of the same order of magnitude. Consider now the case of a circulating droplet at high Pe. Assuming that the external heat transfer is quasi-steady, we can obtain from Equations (4) and (8) the following relation between t_{e_1} and t_{e_2}

$$t_{e_1}/t_{e_2} \simeq 0.033\, Nu_{st}\, \lambda_2/\lambda_1 = 0.033\, Bi_{st} \qquad (9)$$

Let us illustrate this equation with two specific numerical examples for $Pe = 1000$:

1. A single air bubble rises in water at a temperature of ~50°C. Then $\lambda_2/\lambda_1 \simeq 23$; $Nu_{st} \simeq 22$ and $t_{e_1}/t_{e_2} \simeq 16.7$. The heat transfer is controlled by the resistance of the dispersed phase. Note that in this $\rho_2 C_2/\rho_1 C_1 \simeq 4 \cdot 10^3 >> Nu_{st}$, and the external transfer is not quasi-steady. Hence, the value of t_{e_1}/t_{e_2} is even larger than that predicted by (9).

2. A droplet moves in another liquid of similar physical properties: $\lambda_1 \simeq \lambda_2$ and $K \simeq 1$. Then $Nu_{st} = 16.4$ and $t_{e_1}/t_{e_2} \simeq 0.54$. The phase resistances are comparable in magnitude.

If the thermal resistance of the dispersed and continuous phases are comparable, we encounter the so-called "conjugated" problem. In this case, the energy equation should be solved simultaneously for the interior and exterior of the droplet. At the interface, the conditions of continuity for the temperature and the heat flux should be satisfied. Such a problem was analyzed by Levich et al. (1965) and Chao (1969) for a circulating droplet at high Pe. Their solution assumes thin thermal boundary layers on both sides of the interface. The calculated surface temperature of the droplet depends neither on the time nor on the polar angle θ

$$Z_s = 1/(1 + \sqrt{\rho_2 C_2 \lambda_2/\rho_1 C_1 \lambda_1}) \qquad (10)$$

This result coincides with that obtained for two semi-infinite slabs placed suddenly in contact (Gröber et al. 1961). The outer Nusselt number decreases monotonically with time, and at $\tau \gtrsim (1 + K)/Pe$ it approaches its asymptotic value, defined by Eq. (6). The Levich-Chao solution is suitable only at moderate contact times, as long as the internal boundary layer remains thin and the calculated mean dimensionless droplet temperature satisfies the obvious inequality: $\overline{Z}_1 > Z_s$.

Elzinga and Banchero (1959) and Brounshtein and his coworkers (1970, 1976) suggested a different approach to the problem of comparable phase resistances at high Pe. In their model, the internal heat transfer is described by the same partial differential equation of Kronig and Brink (1950) for the circulating droplet, but the boundary condition at the interface takes into consideration the resistance of the continuous phase. The external overall Nusselt number is assumed to be known and equal to its steady value. The Biot number appears in the solution as an additional parameter.

For the second limiting case of $Pe = 0$, the transient heat transfer between a sphere and a surrounding infinite medium was studied analytically by Philip (1964), Brown (1965) and Cooper (1977). In the elementary case of identical thermal properties inside and outside the droplet, the mean dimensionless particle temperature is given by

$$\overline{Z}_1 = 1 - \frac{3}{2\sqrt{\pi}} \int_0^{\tau} \frac{1}{\sqrt{\tau}} [2 - (1 - 2\tau)(1 - e^{-1/\tau})]\, d\tau \qquad (11)$$

For short times $(\tau \leq 0.1)$, this expression may be approximated as $\overline{Z}_1 \simeq 1 - 3\sqrt{\tau/\pi}$, while at long times $(\tau > 2)$, we obtain $\overline{Z}_1 \simeq 1/(6\sqrt{\pi}\tau^{3/2})$. It is interesting that at $\tau \rightarrow \infty$ the Nusselt number does not tend to its well-known steady value $Nu_{st} = 2$, but decreases to 0.

STATEMENT OF THE PROBLEM AND THE PRINCIPAL ASSUMPTIONS

The foregoing analysis shows that the available theoretical studies of the transient conjugate problem pertain only to the extreme cases $Pe = 0$ or $Pe \rightarrow \infty$. In the present paper, the analogous problem is solved for an intermediate region of Peclet numbers $(0 < Pe \leq 1{,}000)$.

Consider a spherical droplet of radius a, moving slowly, for example under the influence of gravity, into an unbound volume of another immiscible liquid. The Reynolds number is assumed to be small $(Re \leq 1)$, and the flow fields inside and outside the droplet are steady-state. We assume that at time $t = 0$ the temperature of the droplet is uniform and is equal to T_0. The initial temperature of the continuous phase is also uniform and is equal to T_∞. The physical properties of the material within the droplet and in the surrounding medium are independent of the temperature. Let (r, θ, ϕ) be the spherical coordinate system, which is fixed with respect to the translating droplet (Figure 1). Here, the 0Z axis coincides with the direction of movement of the droplet, and the angle θ is measured from the frontal stagnation point on the sphere. The velocity and temperature distributions are assumed to be axially symmetric, i.e. $\partial/\partial\phi = 0$.

The unsteady conjugated heat transfer is governed by the following dimensionless energy equations for the interior as well as for the exterior of the droplet

$$\left(\frac{\alpha_2}{\alpha_i}\right)\frac{\partial Z_i}{\partial \tau} + \left(\frac{\alpha_2}{\alpha_i}\right)\frac{Pe}{2}\left(V_{r,i}\frac{\partial Z_i}{\partial r} + \frac{V_{\theta,i}}{r}\frac{\partial Z_i}{\partial \theta}\right) \tag{12}$$
$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial Z_i}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial Z_i}{\partial \theta}\right)$$

Here the subscript i is equal to 1 for the dispersed phase and 2 for the continuous phase. $Z_i = (T_i - T_\infty)/(T_0 - T_\infty)$ is the dimensionless temperature. The velocity components are derived from the Hadamard-Rybczynski stream function (1), (2), thus

$$V_r = -\frac{1}{r^2 \sin\theta}\frac{\partial\psi}{\partial\theta},\ V_\theta = \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r} \tag{13}$$

The dimensionless time $\tau = \alpha_2 t/a^2$ and Peclet number $Pe = 2\,Ua/\alpha_2$ are constructed by employing the physical properties of the continuous phase.

The boundary and initial conditions are:

a) inside the droplet ($0 \le r < 1$)

$$Z_1(r,\ \theta,\ 0) = 1 \tag{14}$$
$$Z_1(0,\ \theta,\ \tau) < \infty \tag{15}$$

b) outside the droplet ($r > 1$)

$$Z_2(r,\ \theta,\ 0) = 0 \tag{16}$$
$$Z_1(\infty,\ \theta,\ \tau) = 0 \tag{17}$$

c) on the droplet surface ($r = 1$)

$$Z_1(1,\ \theta,\ \tau) = Z_2(1,\ \theta,\ \tau) \tag{18}$$
$$\lambda_1\frac{\partial Z_1}{\partial r}(1,\ \theta,\ \tau) = \lambda_2\frac{\partial Z_2}{\partial r}(1,\ \theta,\ \tau) \tag{19}$$

d) on the axis of symmetry ($\theta = 0;\ \pi,\ 0 < r < \infty$)

$$\frac{\partial Z_i}{\partial \theta}(r,\ 0,\ \tau) = \frac{\partial Z_i}{\partial \theta}(r,\ \pi,\ \tau) = 0 \tag{20}$$

The boundary condition (15) indicates that the temperature at the center of the particle must be finite. This requirement results from the choice of a spherical coordinate system.

QUANTITIES OF INTEREST

Solution of Equations (12) with initial and boundary conditions (13) - (20) yields the field of the dimensionless temperature $Z = Z(r,\ \theta,\ \tau)$ for each time τ. The average particle temperature is defined as

$$\overline{Z}_1 = \frac{3}{2}\int_0^1 r^2\left(\int_0^\pi Z_1 \sin\theta\, d\theta\right) dr, \tag{21}$$

and the average surface temperature of the particle as

$$\overline{Z}_s = \frac{1}{2}\int_0^\pi Z_s \sin\theta\, d\theta \tag{22}$$

The overall Nusselt number will be defined as

$$Nu = \frac{2aQ}{4\pi a^2(\overline{T}_1 - T_\infty)\lambda_2} = \frac{-\int_0^\pi\left(\frac{\partial Z_2}{\partial r}\right)_{r=1}\sin\theta\, d\theta}{\overline{Z}_1}, \tag{23}$$

where Q is the overall instantaneous heat flux from the droplet surface. For this definition, the driving force for the heat transfer is taken as the difference between the mixed mean particle temperature, $\overline{T}_1$, and the temperature of the surrounding medium far from the droplet, T_∞. From the heat balance equation for the droplet, the Nusselt number may also be expressed by

$$Nu = -\frac{2}{3}\left(\frac{\rho_1 C_1}{\rho_2 C_2}\right)\frac{d\ln\overline{Z}_1}{d\tau}. \tag{24}$$

The time average Nusselt number is defined over the time interval $(0,\ \tau)$ by

$$\overline{Nu} = \frac{1}{\tau}\int_0^\tau Nu\, d\tau = -\frac{2}{3}\left(\frac{\rho_1 C_1}{\rho_2 C_2}\right)\frac{\ln\overline{Z}_1(\tau)}{\tau} \tag{25}$$

In experimental studies of heat transfer in spray columns, the average Nusselt number is usually calculated using the logarithmic mean difference between the temperatures of the dispersed and the continuous phases. It may be demonstrated that, calculated in this way, $\overline{Nu}$ is identical with (25). In this article we also evaluate the "external" Nusselt number

$$Nu_{ext} = \frac{2aQ}{4\pi a^2(\overline{T}_s - T_\infty)\lambda_2} = \frac{Nu\,\overline{Z}_1}{\overline{Z}_s} \tag{26}$$

and the "internal" Nusselt number

$$Nu_{in} = \frac{2aQ}{4\pi a^2(\overline{T}_1 - \overline{T}_s)\lambda_1} = \frac{Nu\,\overline{Z}_1\lambda_2}{(\overline{Z}_1 - \overline{Z}_s)\lambda_1} \tag{27}$$

By combining Equations (23), (26) and (27) we obtain

$$\frac{1}{Nu} = \frac{\lambda_2}{\lambda_1}\frac{1}{Nu_{in}} + \frac{1}{Nu_{ext}} \tag{28}$$

METHOD OF SOLUTION

We now make the additional assumption that the thermal properties of the dispersed and continuous phases are identical: $\lambda_1 = \lambda_2$ and $\alpha_1 = \alpha_2$ (or $\rho_1 C_1 = \rho_2 C_2$). The assumption is not sufficiently justified even for liquid-liquid systems, in which the quantities (λ_2/λ_1), $(\rho_1 C_1/\rho_2 C_2)$ usually range within the limits of 0.3 to 3. However, this assumption significantly simplifies the solution, since the coefficients in the energy Equations (12) inside and outside of the droplet become identical, and the requirement of continuity of both the temperature and the heat flux, (18), (19) on the droplet surface is automatically satisfied.

We solved this problem numerically by the implicit finite difference method of alternating direction (Richtmyer and Morton 1967). The substitution of the new unknown function $W = rZ$ has made it possible to overcome the difficulties associated with the approximation to the boundary condition (15). For function W, this condition has the simple form $W(0,\ \theta,\ \tau) = 0$. The radial coordinate r for the outer region was replaced by x through the transformation $r = \exp x$. As a result, the use of a constant step size for x made it possible to obtain a more dense mesh near the sphere surface, where the temperature gradients are large, and where an accurate difference approximation is needed. The final equations for the function W may be written as

$$\frac{\partial W}{\partial \tau} + \frac{Pe}{2r^2\sin\theta}\left[-\frac{\partial\psi}{\partial\theta}\left(\frac{\partial W}{\partial r} - \frac{W}{r}\right) + \frac{\partial\psi}{\partial r}\frac{\partial W}{\partial\theta}\right] \tag{29}$$
$$= \frac{\partial^2 W}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 W}{\partial\theta^2} + \frac{\cot\theta}{r^2}\frac{\partial W}{\partial\theta}$$

for the inner region, and

$$e^{2x}\frac{\partial W}{\partial \tau} + \frac{Pe}{2e^x\sin\theta}\left[-\frac{\partial\psi}{\partial\theta}\left(\frac{\partial W}{\partial x} - W\right) + \frac{\partial\psi}{\partial x}\frac{\partial W}{\partial\theta}\right] \tag{30}$$
$$= \frac{\partial^2 W}{\partial x^2} - \frac{\partial W}{\partial x} + \frac{\partial^2 W}{\partial\theta^2} + \cot\theta\frac{\partial W}{\partial\theta}$$

for the external region.

For the numerical solution, the external boundary condition at $r \to \infty$ was closely simulated by satisfying the condition at large but finite distance r_∞ from the center of the sphere, i.e., $W(r_\infty, \theta, \tau) = 0$ (see Woo and Hamielec 1971). This distance was chosen as a function of Pe. Thus, for $Pe = 10$, $r_\infty \simeq 20$ and for $Pe = 1000$, $r_\infty \simeq 5$. The step size for the angular coordinate was uniform and equal to $\Delta\theta = \pi/(M - 1)$, where $M = 31$. The radial step size within the droplet was also taken to be uniform, $\Delta r_1 = 1/(N_1 - 1)$, where $N_1 = 25\text{-}40$. The step size for x was determined from the expression $\Delta x = \ln(1 + \Delta r_1)$ so as to satisfy the condition $\Delta r_1 = \Delta r_2$ in the vicinity of the surface. The number of mesh points in the radial direction for the outer region was $N_2 = \ln r_\infty/\Delta x \simeq 60 - 100$.

To illustrate the finite-difference approximation, we restrict ourselves to Equation (29) for the dispersed phase only. At the first half time step

$$\frac{W_{i,k}^{j+1/2} - W_{i,k}^{j}}{\Delta\tau/2} = \frac{Pe}{2r_i^2\sin\theta_k}\frac{\delta\psi}{\delta\theta}$$

$$\left[\frac{W_{i+1,k}^{j+1/2} - W_{i-1,k}^{j+1/2}}{2\Delta r_1} - \frac{W_{i,k}^{j+1/2}}{r_i}\right]$$

$$+\frac{W_{i+1,k}^{j+1/2} - 2W_{i,k}^{j+1/2} + W_{i-1,k}^{j+1/2}}{\Delta r_1^2} + \frac{1}{r_i^2}\frac{\delta^2 W^j}{\delta\theta^2}$$

$$+\frac{1}{r_i^2\sin\theta_k}\frac{\delta W^j}{\delta\theta}\left(\cos\theta_k - \frac{Pe}{2}\frac{\delta\psi}{\delta r}\right) \quad (31)$$

At the second half time step:

$$\frac{W_{i,k}^{j+1} - W_{i,k}^{j+1/2}}{\Delta\tau/2} = \frac{Pe}{2r_i^2\sin\theta_k}\frac{\delta\psi}{\delta\theta}\left[\frac{\delta W^{j+1/2}}{\delta r} - \frac{W_{i,k}^{j+1/2}}{r_i}\right]$$

$$+\frac{\delta^2 W^{j+1/2}}{\delta r^2} + \frac{1}{r_i^2}\left[\frac{W_{i,k+1}^{j+1} - 2W_{i,k}^{j+1} + W_{i,k+1}^{j+1}}{\Delta\theta^2}\right. \quad (32)$$

$$\left. + \left(\cot\theta - \frac{Pe}{2\sin\theta}\frac{\delta\psi}{\delta r}\right)\frac{(W_{i,k+1}^{j+1} + W_{i,k-1}^{j+1})}{2\Delta\theta}\right]$$

Here, the subscripts i,k designate the spatial mesh position ($2 \le i \le N_1$, $2 \le k \le M - 1$), while the superscript denotes the time for which the value of function W is taken: j is the time τ; $(j + 1/2)$ is the time $(t + \Delta\tau/2)$; $(j + 1)$ is the time $(\tau + \Delta\tau)$; $r_i = (i - 1)\cdot\Delta r$; $\theta_k = (k - 1)\cdot\Delta\theta$; symbols $\delta/\delta r$, $\delta/\delta\theta$, $\delta^2/\delta r^2$, $\delta^2/\delta\theta^2$ define the second-order accuracy finite-difference analogues of the partial derivatives, i.e.,

$$\frac{\delta W}{\delta r} = \frac{W_{i+1,k} - W_{i-1,k}}{2\Delta r_1}\,;\ \frac{\delta^2 W}{\delta r^2} = \frac{W_{i+1,k} - 2W_{i,k} + W_{i-1,k}}{\Delta r_1^2}$$

The finite-difference approximation for the outer region may be written analogously. The boundary conditions at $r = 0$ and $r = r_\infty$ are

$$\left.\begin{aligned} W(1,k) &= 0 \\ W(N_1 + N_2,\ k) &= 0 \end{aligned}\right\} \text{at all } \tau \text{ and } 1 \le k \le M$$

Symmetry conditions (20) are approximated with second-order accuracy

$$3\,W_{i,1} = 4\,W_{i,2} - W_{i,3}\,;\ 3\,W_{i,M} = 4\,W_{i,M-1} - W_{i,M-2}$$
$$(1 < i < N_1 + N_2)$$

The initial condition for function W is

$$W_{i,k} = \begin{cases} r_i & (i = 1,2,\ldots,N_1 - 1) \\ 0.5 & (i = N_1) \\ 0 & (i = N_1 + 1,\ldots,N_1 + N_2) \end{cases}$$

The initial temperature on the droplet surface was taken to be $Z_s = 0.5$ which corresponds to the analytical solution of the problem of heat conduction for very small τ (see Cooper 1977, Chao 1969).

Time step $\Delta\tau$ was variable and usually ranged from $\Delta\tau = 10^{-6}$ at the start of the computations to 10^{-2} at the final stage.

An analogous finite-difference scheme has previously been successfully employed for solving the external problem of convective heat and mass transfer for intermediate values $Pe \lesssim 1000$ (Abramzon et al. 1976).

RESULTS AND DISCUSSION

For the limiting case of $Pe = 0$, our results are in excellent agreement with the exact solution of Cooper (1977). For intermediate Pe, most of the computations were made at two values of the relative viscosity: $K = 1$ (droplet with internal circulation) and $K = \infty$ (rigid particle or stagnation droplet). Figures 2a, b, c show the development in time of the temperature fields within and outside of the particle at different Pe and K. The numbers on the curves represent the values of the dimensionless temperature Z. At short times, τ, the radial temperature gradients near the surface of the sphere are high, and heat is transferred primarily by conduction. The shape of the isotherms is almost spherical, particularly within the droplet, where the convective

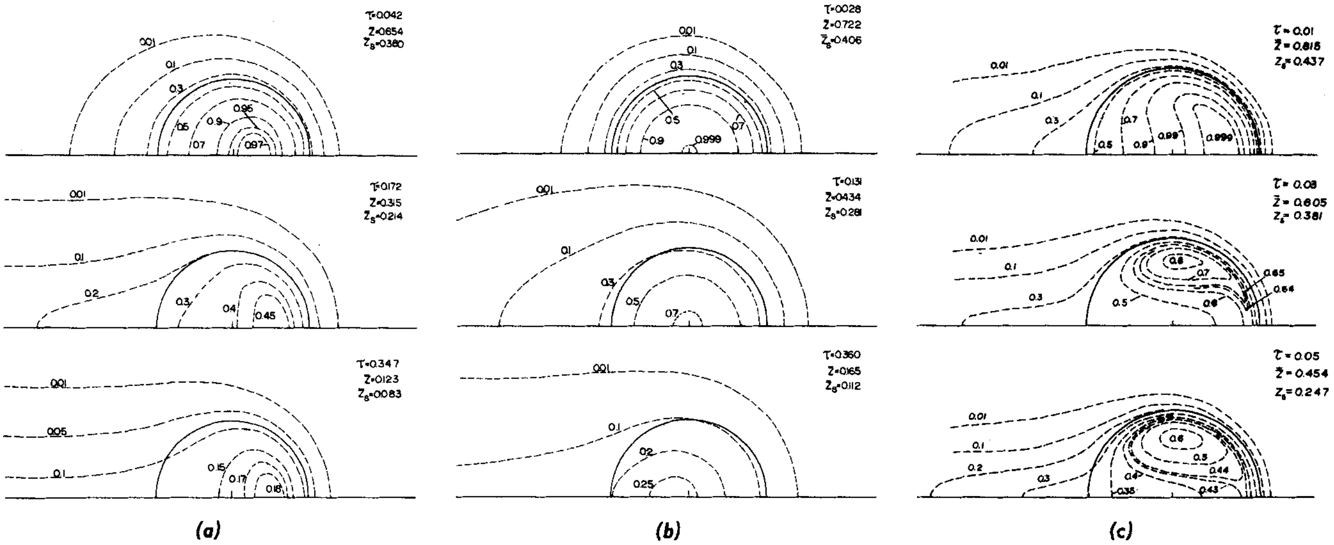

Figure 2. Temporal development of the temperature field within and outside the particle: a) $Pe = 100$, $K = 1$ (droplet with internal circulation); b) $Pe = 100$, $K = \infty$ (rigid particle); and c) $Pe = 1000$, $K = 1$ (droplet with internal circulation).

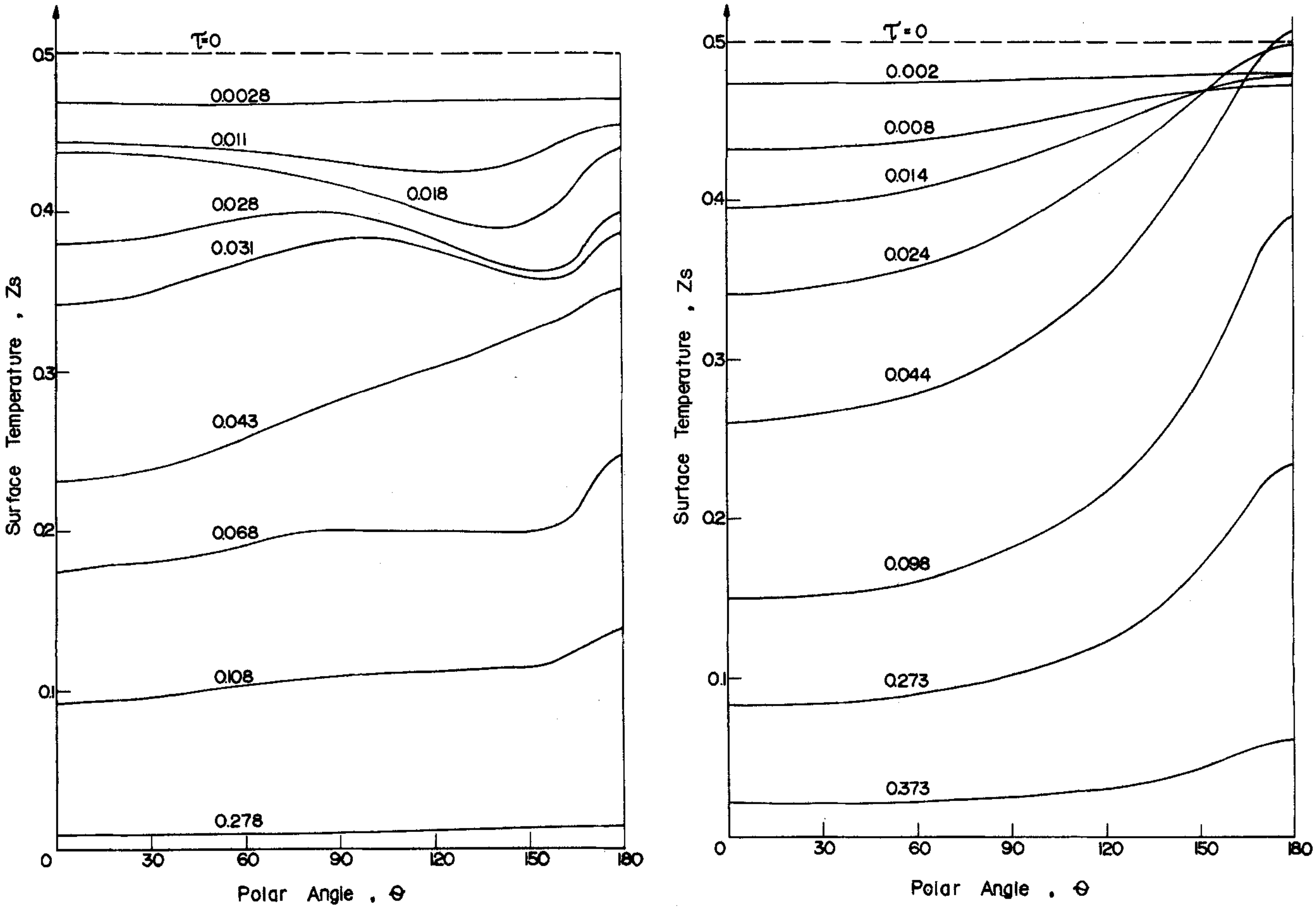

Figure 3. Surface temperature as a function of time and of angle from frontal stagnation point ($Pe = 1000$): a) $K = 1$ (droplet with internal circulation); b) $K = \infty$ (rigid particle).

effects are weaker than in the outer region. At large times, τ, convective transport has the dominant role, and the isotherms become deformed under the influence of liquid flow. For $r > 1$ the temperature distribution is similar to that for the external problem (Abramzon et al. 1976, 1977).

The shape of isotherms inside the particle is strongly dependent on Pe and K. Within a rigid particle the isotherms are somewhat shifted from the center to the rear zone. The reason for this shift is that the rate of cooling over the front part of the surface is much higher than that for the rear part.

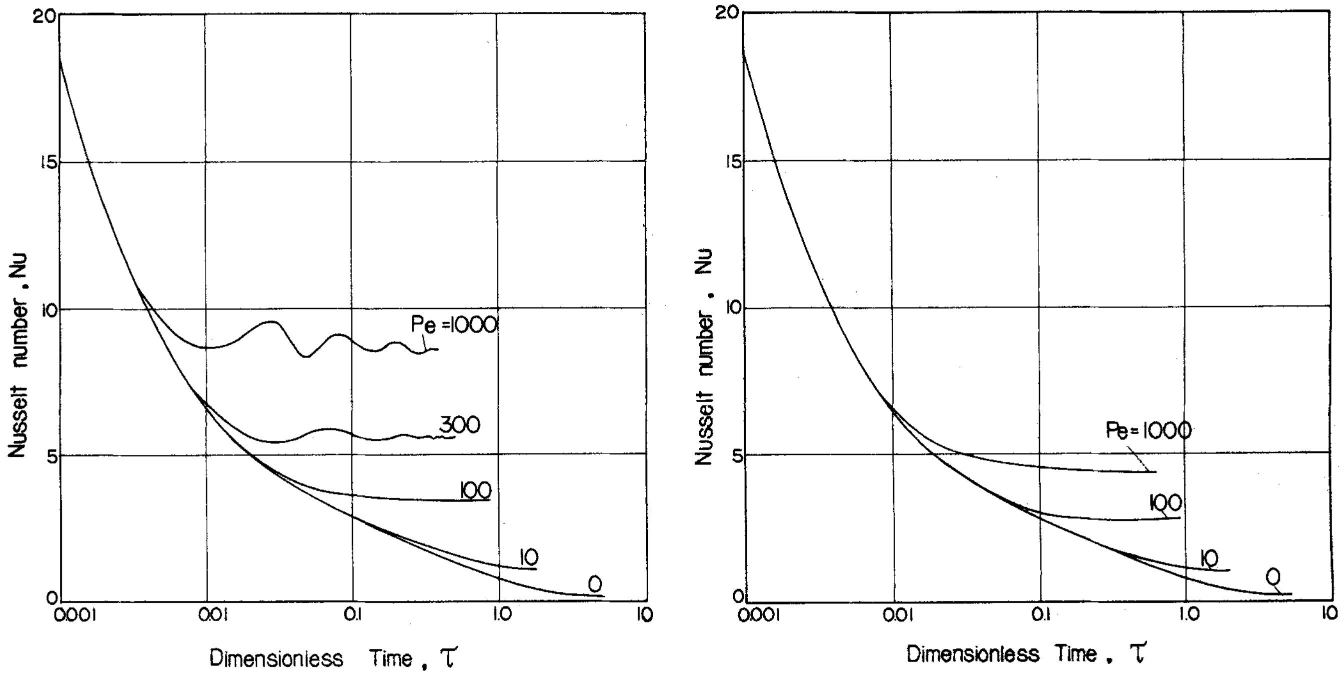

Figure 4. Nusselt number at various Pe as a function of time: a) $K = 1$ (droplet with internal circulation); b) $K = \infty$ (rigid particle).

TABLE 1. COMPARISON OF THE ASYMPTOTIC NUSSELT NUMBER WITH ITS ESTIMATES BASED ON EQUATIONS (33), (37).

K		Pe			
		0	10	100	1000
1	Steady-state Nusselt number Nu_{st} or the external problem (Abramzon et al., 1976, 1977)	2.0	3.6	6.8	16.4
	Asymptotic value of external Nusselt number Nu_{ext} (present study)	0	1.2	5.2	16.3
	Asymptotic value of Nu_{in} for the internal problem (Johns and Beckmann, 1965)	6.58	$\simeq 7$	$\simeq 10.5$	17.9
	Asymptotic value of Nu^* (equation 33)	1.53	2.37	4.13	8.56
	Asymptotic value of Nu^* (present study)	0	1.1	3.5	8.5
∞	Steady-state Nusselt number Nu_{st} for the external problem (Abramzon et al., 1976)	2.0	3.15	5.38	10.6
	Asymptotic value of external Nusselt number Nu^*_{ext} (present study)	0	1.02	4.10	10.4
	Asymptotic value of Nu^*_{in} for the internal problem (Newman, 1931)	6.58	6.58	6.58	6.58
	Asymptotic value of Nu^* (equation 33)	1.53	2.13	2.96	4.05
	Asymptotic value of Nu^* (equation 37)	0	0.87	2.59	4.33
	Asymptotic value of Nu^* (present study)	0	0.92	2.85	4.4

In the case of a circulating droplet, the internal motion of the liquid distorts the isotherms and shifts them toward the front part. At $Pe \leq 100$, the temperature maximum lies on the axis of symmetry, while at higher Pe some of the isotherms take the shape of toroidal surfaces, which are similar to the streamlines. Qualitatively, the temperature distributions inside a rigid sphere or a droplet with full circulation agree with the experimental observations of Head and Hellmus (1966).

In Figures 3a, b the temperature distribution along the surface of the sphere, $Z_s(\theta)$, is shown for $Pe = 1,000$ for different times. It can be seen that the surface temperature changes significantly over the entire cooling period, unlike the Levich-Chao solution (10). The temperature drop along the surface of the rigid sphere (Figure 3b) is larger than that for the droplet (Figure 3a), because the internal circulation aids in reducing the temperature gradients within the droplet.

The time dependence of the instantaneous Nusselt number, $Nu = Nu(\tau)$, for a droplet with internal circulation and for a rigid sphere at different Pe is depicted in Figures 4a, b. At small times, τ, all of the curves coincide with the solution for $Pe = 0$. This fact points to the predominant contribution of conduction to the heat transfer. The effect of convective transfer becomes perceptible after some contact time τ, which depends on Pe. At $\tau \rightarrow \infty$, the Nusselt number approaches some asymptotic value Nu^*, which is found to be significantly lower than both the asymptotic value for the internal problem, Nu^*_{in}, and the steady-state value for the external problem Nu_{st} (Table 1). For a droplet with a full circulation at $Pe \gtrsim 300$, damping oscillations of the instantaneous Nu are observed (Figure 4a). This result agrees with published data cited above, The frequency of these oscillations increases with Pe. This phenomenon causes serious difficulties in solving the problem by finite-difference methods. For the correct numerical solution, the time step, $\Delta\tau$, must be much smaller than the period of oscillation. Hence, the required computation time increases rapidly with an increase in Pe.

The behavior of the external Nusselt number, $Nu_{ext}(\tau)$, is analogous to the curves of $Nu(\tau)$ which have just been examined. As τ increases, Nu_{ext} approaches some asymptotic value, Nu^*_{ext}, which was found to be lower than the corresponding steady-state value of Nu_{st} for the external problem. This divergence is especially significant at moderate Peclet numbers ($Pe < 10 \div 100$) while at $Pe \geq 1000$ the values of Nu^*_{ext} and Nu_{st} are practically identical (Table 1).

A possible physical explanation of this effect of Pe on Nu^*_{ext} may be offered. At low Pe, the time of formation of the external boundary layer and the characteristic cooling times are comparable ($\tau_r \simeq \tau_e$). The temperature field near the particle does not completely succeed in reconstructing itself to follow the reduction in surface temperature, T_s. Hence the temperature gradient near the surface is lower than that for the steady-state case. As Pe rises, the relaxation time τ_r drops steeply. Thus, for a rigid particle (Abramzon et al., 1976, and Figure 5b of the present paper), we have: $\tau_r \simeq 10$, $\tau_e \simeq 0.17$ at $Pe = 0$, while $\tau_r \simeq 0.02$ and $\tau_e \simeq 0.12$ at $Pe = 1000$. These data indicate that $\tau_r << \tau_e$ at high Pe, and hence the external heat transfer may be treated as quasi-steady, i.e., $Nu^*_{ext} \simeq Nu_{st}$.

For rough estimation of the asymptotic Nusselt number Nu^* at high Pe the approximate formula

$$\frac{1}{Nu^*} \simeq \frac{\lambda_2}{\lambda_1} \cdot \frac{1}{Nu^*_{in}} + \frac{1}{Nu_{st}} \qquad (33)$$

can be used. Here Nu^*_{in} is the asymptotic value of the Nusselt number for the *internal* problem, which is determined from data of Johns and Beckmann (1966) and Newman (1931).

For the case of a rigid sphere, the asymptotic Nusselt number Nu^* may be predicted from the simplified one-dimensional "film model." Assume that the local heat transfer coefficient does not depend on the angle θ. Then, the temperature is a function only of the coordinate r and of the time τ. The heat transfer in the continuous phase may be simulated by introducing some "effect stagnation liquid film" around the particle. The film thickness is calculated from the assumption that in the steady-state case, the rate of conductive transport through the film is equal to the actual rate of the external heat transfer. This yields the following expression for the external radius of the film

$$r_f = \frac{Nu_{st}}{Nu_{st} - 2} \qquad (34)$$

We shall now assume that at $\tau \rightarrow \infty$, the temperature at every point in the particle and in the film may be written as $Z =$

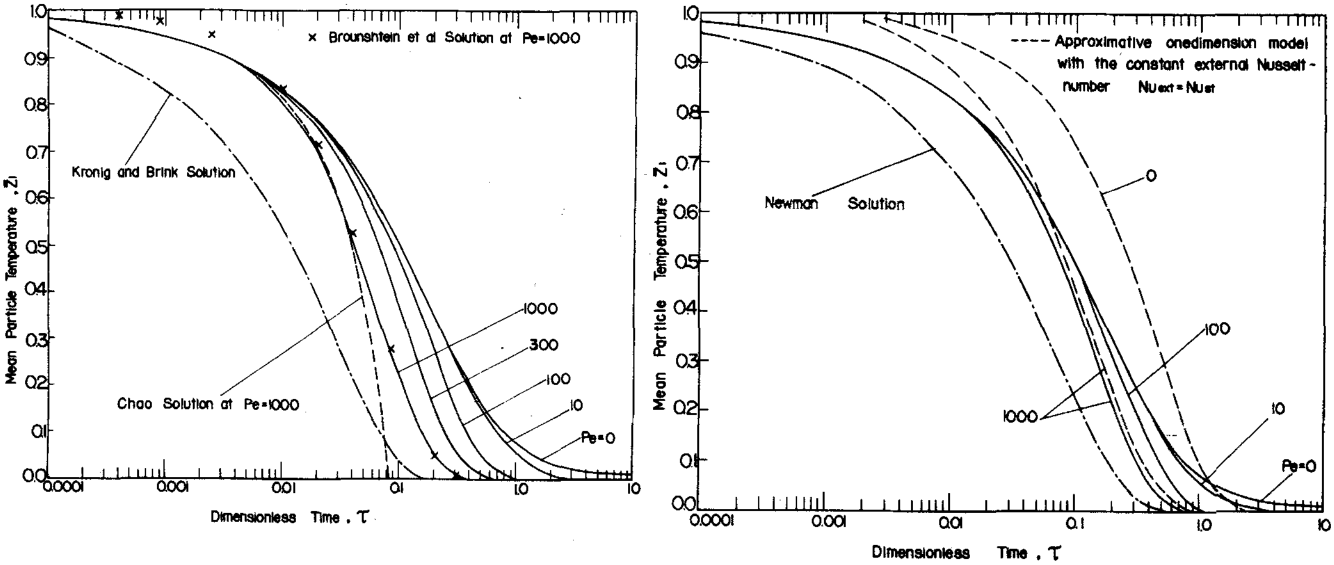

Figure 5. Average particle temperature as a function of time and Peclet number. Comparison of the present numerical solution with the various approximative models: a) $K = 1$ (droplet with internal circulation); b) $K = \infty$ (rigid particle).

$f_i(r)exp(-\beta\tau)$, where $f_i(r)$ is a function depending only on r. Placing this expression into the unsteady heat conduction equations both for the particle and for the film, we obtain

$$Nu^* = \frac{2}{3}\left(\frac{\rho_1 C_1}{\rho_2 C_2}\right)\beta, \tag{35}$$

where β is the first positive root of the equation

$$\frac{\lambda_1}{\lambda_2}\left[\sqrt{\alpha_2\beta/\alpha_1}\cot\sqrt{\alpha_2\beta/\alpha_1} - 1\right] = \sqrt{\beta}\cot\sqrt{\beta}(1 - r_f) - 1 \tag{36}$$

In the simple case of identical physical properties we have

$$Nu^* = \frac{2}{3}\pi^2\left(\frac{Nu_{st} - 2}{Nu_{st}}\right)^2 \tag{37}$$

In Table 1, the values of Nu^* calculated by the present finite-difference method are compared with those estimated from Equations (33) and (37). As can be seen, for the rigid sphere Equation (37) gives more precise values than Equation (33).

Figure 5 shows the variation in the average temperature of a droplet and of a rigid particle as a function of time τ and of Pe. The dash-and-dot curves in these figures depict the solutions of Kronig and Brink (1950) and Newman (1931) for the case in which heat transfer is governed by the resistance of the dispersed phase. These solutions yield the upper limit of the heat transfer rate. The curve for $Pe = 0$ is the lower limit.

The dashed curve in Figure 5a represents the Levich-Chao solution for $Pe = 1000$. The accuracy of this solution is satisfactory only at short times, as long as $\bar{Z}_1 > 0.5$. As opposed to this, the solution of Brounshtein et al. (1970) is in satisfactory agreement with the exact numerical results at $Pe = 1000$ for nearly all values of τ, except for those in the short initial region. At small τ, the actual instantaneous external Nusselt number is much greater than the value which was used in the model of Brounshtein et al. (1970) ($Nu_{ext} >> Nu_{st}$). For this reason, the use of the model of Brounshtein et al. (1970) leads to an underestimation of the rate of heat transfer during the initial stage of cooling.

For a rigid sphere, the heat transfer is often calculated from the one-dimensional model, assuming the external Nusselt number to be constant and equal to its steady-state value ($Nu_{ext} = Nu_{st}$). The solution of this classical problem is presented, for example, in the textbook of Gröber et al. (1961) in the form of graphs of $\bar{Z}_1 = \bar{Z}_1(\tau, Bi)$. For high Pe, this model yields satisfactory results for $\bar{Z}_1$ during the final stages of cooling, while for low Pe, significant divergence from the exact solution is observed at almost all values of τ (see Figure 5b).

One of the most interesting results of the present work is that the asymptotic Nusselt numbers Nu^* and Nu^*_{ext} at moderate Pe are far below the corresponding steady-state value Nu_{st}. It should be noted that similar "anomalously" low heat transfer coefficients were observed experimentally for liquid-liquid spray columns (see, for example, Pierce et al. 1959, Greskovich et al. 1967). We consider that the analysis developed above may provide a partial explanation for this phenomenon.

In conclusion, note that the results of the present investigation of the phenomenon of heat transfer are also applicable to mass transfer in the case in which the distribution coefficient is approximately equal to 1.

NOTATION

a = sphere radius
Bi = Biot number
C = specific heat
K = viscosity ratio, $K = \mu_1/\mu_2$
M = number of equal increments in the angular direction
N = number of steps in the radial direction
Nu = overall Nusselt number defined by Equation (23) or (24)
Nu_{in} = "internal" Nusselt number defined by Equation (27)
Nu_{ext} = "external" Nusselt number defined by Equation (26)
Nu_{st} = steady-state value of the Nusselt number for the external problem
Pe = Peclet number, $Pe = 2Ua/\alpha_2$
Q = overall heat flux from the spherical surface
R = distance from the center of sphere
Re = Reynolds number, $Re = 2Ua\rho_2/\mu_2$
r = dimensionless radial coordinate, $r = R/a$
r_∞ = position of outer, concentric, spherical boundary in the numerical solution
T = temperature
t = time
t_e = characteristic time of cooling (heating) of the sphere
t_r = relaxation time of the external temperature field
U = free-stream fluid velocity
V_r = radial velocity component
V_θ = tangential velocity component
W = auxiliary unknown function, $W = rZ$
x = modified radial spherical coordinate for the external region, $x = \ln r$
Z = dimensionless temperature, $Z = (T - T_\infty)/(T_0 - T_\infty)$

Greek Letters

α = thermal diffusivity
β_1 = first positive root of Equation (36)

Δr = step size in radial direction
$\Delta\theta$ = angular step size
$\Delta\tau$ = time step size
θ = polar angle, spherical coordinate system
λ = thermal conductivity
μ = dynamic viscosity
ρ = density
τ = dimensionless time, $\tau = \alpha_2 t/a^2$
ψ = stream function

Subscripts

i,k = mesh point coordinates
0 = initial condition
1 = refers to disperse phase (interior of the droplet)
2 = refers to continuous phase (exterior of the droplet)
∞ = refers to large distance from the spherical particle

Superscripts

$*$ = asymptotic value (for $\tau \to \infty$)
$-$ = volume- or surface-averaged value
$j, j+1/2, j+1$ correspond to time τ, $(\tau + \Delta\tau/2)$, and $(\tau + \Delta\tau)$, respectively

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Emulsion Phase Residence Time and Its Use in Heat Transfer Models in Fluidized Beds

Heat transfer from a vertical tube in fluidized beds was investigated by measurement of emulsion packet residence times. The root-square-mean residence times were then used in a modified packet model to successfully predict effective heat transfer coefficients.

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SCOPE

Many theoretical models have been developed for estimating the heat transfer coefficient to tubes submerged in fluidized beds. One of the oldest and most widely accepted is the packet renewal model originally developed by Mickley and Fairbanks (1955, 1961). In this, as in many of the other models, the heat transfer process is postulated to be governed by the residence time of the emulsion phase (packets) on the heat transfer sur-

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